MATHCOUNTS

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Aiden and Bryce are racing around a race track. They begin together at the starting line, and Aiden's car completes a lap every 44 seconds, while Bryce's car completes a lap every 40 seconds. How many seconds after they begin the race will Aiden and Bryce first reach the starting line at the same time?

Let a and b be the number of laps that Aiden and Bryce, respectively, make when they meet again at the starting line. Because the requisite meeting place is at the starting line, that involves a whole number of laps for each, thus making both a and b to be integers. The times involved are $44a$ seconds and $44b$ seconds, which must be equal to be a meeting. Now, $44a$ seconds = $40b$ seconds, and manipulating algebraically yields $b/a = 44/40 = 11/10$, which cannot be reduced further. Therefore, Bryce takes 11 laps while Aiden takes 10 laps, corresponding to 10 x 44 seconds = 11 x 40 seconds = 440 seconds.

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Deb is watching an online movie with a playing time of 2 hours and 18 minutes, not including ads. The website shows a 30-second ad prior to starting the movie. The website interrupts the movie after each 10 minutes that the movie has played and shows another 30-second ad. If Deb watches from the beginning of the first ad to the end of the movie, what percent of her total viewing time does she spend watching ads? Express your answer as a decimal to the nearest tenth.

The movie lasts 2 hours + 18 minutes = 2 x 60 minutes + 18 minutes = 120 minutes + 18 minutes = 138 minutes. An ad is shown at the very beginning and then after each 10 minutes of a movie, thus at 0, and after 10 minutes, 20 minutes, …, 130 minutes of a movie, which is a total of 14 ads. Each ad takes 30 seconds, which is (1/2) minute, so 14 of them last 14 x (1/2) minutes = 7 minutes. Therefore, the total watching time is 138 minutes + 7 minutes = 145 minutes, of which 7 minutes is advertising, making the fraction of time spent watching ads equal to 7 minutes/(145 minutes) = (7/145) x 100% ~ 4.8%.

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For a computer code, nine numbers are grouped into three sets of three numbers, with one number in each set designated as a key. The nine numbers are 2, 3, 5, 7, 11, 13, 17, 19, and 29. If they must be used according to the eight rules, what is the product of the three numbers in the set that does not contain 2 or 3?

Computer Code Rules	
2 and 3 must be keys.	13 must be with 11 or 19.
5 and 7 must be in the same set; neither is a key.	7 cannot be in a set with 17 as a key.
13, 17, and 29 must be in different sets.	13 cannot be in a set with 2 as a key.
11 and 19 must be in different sets.	19 cannot be in a set with 29 as a key.

We know that 2 and 3 are keys, and there is a third, as yet unknown key. Because we want the product of all three elements in the set that contains neither 2 nor 3, that unknown key must be determined.

5 and 7 go in the same set, but neither is a key, so those two values go with one of the three keys, and that key is the only thing that is unknown for that set.

Because 13, 17, and 29 must be in different sets, one of them must be the key in the set with 5 and 7.

Now, 13 must be with 11 or 19, which cannot be the case if it is with 5 and 7, so we are down to 17 and 29 for consideration.

7 cannot be in a set with 17 as a key, and we are dealing with the set with 5 and 7, so 17 cannot be the key, so the only value left for the key is 29.

Our set appears to be {29, 5, 7}. A quick check of the other rules yields no contradiction. The desired answer is the product 29 x 5 x 7 = 1015.